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Scalar potential for the gauged Heisenberg algebra and a non–polynomial antisymmetric tensor theory

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Abstract

We study some issues related to the effective theory of Calabi–Yau compactifications with fluxes in Type II theories. At first the scalar potential for a generic electric abelian gauging of the Heisenberg algebra, underlying all possible gaugings of RR isometries, is presented and shown to exhibit, in some circumstances, a "dual" no–scale structure under the interchange of hypermultiplets and vector multiplets. Subsequently a new setting of such theories, when all RR scalars are dualized into antisymmetric tensors, is discussed. This formulation falls in the class of non–polynomial tensor theories considered long ago by Freedman and Townsend and it may be relevant for the introduction of both electric and magnetic charges.

1. Introduction

In a recent paper [1] we have proposed a representation of Calabi–Yau compactifications with fluxes as gauged supergravity where the gauge group is embedded in the Heisenberg algebra of the $2\,h_1+3$ isometries [2, 3, 4] (recall that according to our conventions $h_1=h_{11}$, $h_2=h_{12}$ in Type IIB setting while $h_1=h_{12}$, $h_2=h_{11}$ in Type IIA) corresponding to the R–R scalars and the scalar field a dual to the NS 2–form $B_{\mu\nu}$. The Heisenberg algebra has the following symplectic structure:

$$[X_A,\,X_B] \;=\; -2\,\mathbb{C}_{AB}\,\mathscr{Z}\,, \tag{1}$$

where $X_A=\{X^\Lambda,\,X_\Lambda\}$, $\Lambda=1,\dots,h_1+1$ and $\mathbb{C}_{AB}$ is the symplectic invariant matrix: $\mathbb{C}^\Lambda{}_\Sigma=-\mathbb{C}_\Sigma{}^\Lambda=\delta^\Lambda{}_\Sigma$, $\mathbb{C}_{\Lambda\Sigma}=\mathbb{C}^{\Lambda\Sigma}=0$. For the sake of simplicity it will be useful in the sequel to extend the index A to an index $m=\{0,A\}=0,\dots,h_1+1$, so that, defining $X_0=\mathscr{Z}$, equation (1) can be rewritten in the form $[X_m,\,X_n]=f_{mn}{}^p\,X_p$, where the only non vanishing components of the structure constants are: $f_{AB}{}^0=-2\,\mathbb{C}_{AB}$.

It is straightforward to see that gauging this algebra by h_2 vectors corresponds to defining the following gauge generators:

$$T_I \;=\; e_I{}^A\,X_A+c_I\,\mathscr{Z}=e_I{}^m\,X_m\,, \tag{2}$$

where $e_I{}^0=c_I$ and $I=0,\dots,h_2$. The gauge algebra $\{T_I\}$ closes within the Heisenberg algebra, with structure constants $f_{IJ}{}^K$, namely $[T_I,\,T_J]=f_{IJ}{}^K\,T_K$, only if the following conditions are satisfied:

$$\begin{aligned} e_I{}^m\,e_J{}^n\,f_{mn}{}^p &= f_{IJ}{}^K\,e_K{}^p\,,\\ f_{IJ}{}^K\,e_K{}^A &= 0\,. \end{aligned} \tag{3}$$

In what follows we shall need to restrict ourselves to an abelian gauge algebra, namely $[T_I,\,T_J]=0$. In this case the “electric” charges $e_I{}^m$ satisfy the trivial–cocycle condition [1]:

$$e_I{}^m\,e_J{}^n\,f_{mn}{}^k \;=\; -2\,e_I{}^A\,e_{JA}=0\,. \tag{4}$$

The $N=2$ scalar potential for such general electric gauging is then constructed using the standard formulae of $N=2$ supergravity [5]. On the other hand if the scalar a is dualized into the 2–form $B_{\mu\nu}$, the residual algebra associated to the generators X_A becomes abelian: $[X_A,\,X_B]=0$. This reflects the fact that in the Lagrangian all the scalar fields $V_A=\{\zeta^\Lambda,\,\tilde\zeta_\Lambda\}$ parametrizing X_A appear covered by derivatives and the coupling to the B field

$$dV_A\wedge dV^A\wedge B\,, \tag{5}$$

is invariant under $V_A \to V_A + \text{const.}$ and $B \to B + d\Lambda$.

When the Lagrangian is written in such form one can use Poincaré duality between 1 and 3-forms in four dimensions to obtain a dual Lagrangian which involves tensor fields $B_m = \{B^\Lambda, B_\Lambda, B_0\}$, where $B_0 = B$, B_Λ are dual to ζ^Λ and B^Λ dual to $\tilde{\zeta}_\Lambda$. The coupling (5) gives rise to a non–polynomial theory when Poincaré duality is used and this reflects the non–abelian structure of the original Heisenberg algebra written in (1). This makes contact with an old work by Freedman and Townsend [6] where a non–abelian tensor gauge theory was formulated and also the coupling to gauge fields (associated to a semisimple gauge group) discussed.

The paper is organized as follows:

In section 2 we exhibit the form of the scalar potential for a generic electric (abelian) gauging of the Heisenberg algebra and obtain a structure which, for some particular choice for the electric fluxes, reproduces the scalar potential derived in the context of Type IIA,B compactifications [7, 8, 9, 10, 11, 12, 13, 14] on *half–flat* manifolds with NS fluxes [16, 15, 17].

A dual structure is found in the potential when the Heisenberg algebra of the special quaternionic isometries is gauged. This allows to prove, as an example, that in the case in which ${e_I}^\Lambda = 0$ the potential for $e_{I\Lambda} = e_{I0}$, with cubic special geometry for the vector sector, has the same no–scale structure as the potential for $e_{I\Lambda} = e_{0\Lambda}$ and cubic special geometry for the hypermultiplets. This allows to prove the equivalence between compactifications on a half–flat Calabi–Yau manifold and compactifications in the presence of NS fluxes on the mirror manifold. The antisymmetric tensor formulation should allow to describe theories were both electric and magnetic charges are introduced [18, 19, 20], thus realizing a completely $\mathrm{Sp}(2\,h_2+2)\times\mathrm{Sp}(2\,h_1+2)$ duality invariant scalar potential, which contain all the previous examples as particular cases.

2. Scalar potential with electric fluxes

Let us start by introducing some notations. The scalars of a special quaternionic manifold are denoted by:

$$q^u = \{\phi,\, a,\, \zeta^\Lambda,\, \tilde{\zeta}_\Lambda,\, Z^\Lambda\} \;\; ; \;\; \Lambda = 0,\dots,h_1\,, \tag{6}$$

where, from Type IIB point of view, a is the scalar dual to the 2–form NS tensor $B_{\mu\nu}$, $\zeta^0 = C_{(0)}$, $\zeta^\Lambda = C^\Lambda_{(2)}$ $(\Lambda \neq 0)$, $\tilde{\zeta}_0$ is dual to $C_{\mu\nu}$, $\tilde{\zeta}_\Lambda = C_{(4)\,\Lambda}$ $(\Lambda \neq 0)$, ϕ is the four–dimensional dilaton and Z^Λ is the projective special coordinate vector describing the remaining NS moduli z^a $(a = 1,\dots,h_1)$: $Z^0 = 1$, $Z^a = z^a$. As anticipated in the introduction, there exists a subgroup of the isometry group generated by a Heisenberg

algebra $\{X^\Lambda,\, X_\Lambda,\, \mathscr{Z}\}$, whose action of the hyperscalars has the following form:

$$\begin{aligned} \delta\zeta^\Lambda &= \alpha^\Lambda\,, \\ \delta\tilde{\zeta}_\Lambda &= \beta_\Lambda\,, \\ \delta a &= \gamma + \alpha^\Lambda \tilde{\zeta}_\Lambda - \beta_\Lambda \zeta^\Lambda\,, \end{aligned} \tag{7}$$

and whose structure is[1]:

$$\begin{aligned} \left[X^\Lambda,\, X_\Sigma\right] &= -2\,\delta^\Lambda{}_\Sigma\,\mathscr{Z}\,, \\ \left[X^\Lambda,\, X^\Sigma\right] &= \left[X_\Lambda,\, X_\Sigma\right] = \left[X^\Lambda,\, \mathscr{Z}\right] = \left[X_\Lambda,\, \mathscr{Z}\right] = 0\,. \end{aligned} \tag{8}$$

The gauge generators T_I ($I = 0, \dots, h_2$) have the following form in terms of the Heisenberg isometries:

$$T_I = e_I{}^\Lambda\, X_\Lambda - e_{I\Lambda}\, X^\Lambda + c_I\, \mathscr{Z}\,, \tag{9}$$

and the corresponding Killing vectors are:

$$k_I = (c_I + e_I{}^\Lambda\, \tilde{\zeta}_\Lambda - e_{I\Lambda}\, \zeta^\Lambda)\, \frac{\partial}{\partial a} + e_I{}^\Lambda\, \frac{\partial}{\partial \zeta^\Lambda} + e_{I\Lambda}\, \frac{\partial}{\partial \tilde{\zeta}_\Lambda}\,. \tag{10}$$

The general form of the $\mathscr{N} = 2$ scalar potential is [5]:

$$\mathscr{V} = 4\, h_{uv} k^u_I k^v_J\, L^I\, \overline{L}^J + g_{i\bar{\jmath}}\, k^i_I k^{\bar{\jmath}}_J\, L^I\, \overline{L}^J + (U^{IJ} - 3\, L^I \overline{L}^J) \mathscr{P}^x_I\, \mathscr{P}^x_J\,, \tag{11}$$

where L^I are the upper components of the covariantly holomorphic symplectic section of the special Kähler manifold, k^u_I and k^i_I are respectively the quaternionic and special Kähler Killing vectors and U^{IJ} denotes the matrix [5]:

$$U^{IJ} = -\frac{1}{2}\, \mathrm{Im}(\mathscr{N})^{-1\,IJ} - \overline{L}^I\, L^J = D_i L^J\, D_{\bar{\jmath}} \overline{L}^J\, g^{i\bar{\jmath}}\,, \tag{12}$$

h_{uv}, $g_{i\bar{\jmath}}$ being the metrics of the hyper– and vector multiplet geometries. The second term in (11) does not contribute to the gauging we are considering, which involves quaternionic isometries only. The expression for the momentum maps $\mathscr{P}^x_J$ is [22, 23]:

$$\mathscr{P}^x_I = k^u_I\, \omega^x_u\,, \tag{13}$$

where ω^x is the SU(2) connection. This form is Heisenberg–invariant and so is therefore the SU(2) curvature. This justifies the absence of a compensator on the right hand side of eq. (13).

[1]Note that, when extending the Heisenberg algebra with the Cartan generator corresponding to the ϕ coordinate, one obtains the $2\,h_1 + 4$ dimensional solvable algebra generating the symmetric coset $\mathrm{SU}(1, 2 + h_1)/\mathrm{U}(1) \times \mathrm{SU}(2 + h_1)$ [21]. This is the minimal number of isometries of a special quaternionic manifold.

The NS scalars Z^Λ parametrize a special Kähler submanifold of the quaternionic manifold. It is useful to characterize this manifold in terms of a Kähler potential K, a prepotential $\mathscr{F}$ and period matrix [2]:

$$\mathcal{M} \;=\; i\,\overline{\mathscr{N}}_s\,, \tag{14}$$

where $\mathscr{N}_s$ is the period matrix as defined in [2] and $\mathcal{M}$ is the matrix used in e.g. [16] and K being the Kähler potential associated with it. Let us define the following forms:

$$\begin{aligned}
v &= e^{\tilde{K}}\,[d\phi - i\,(da + \tilde{\zeta}^T\,d\zeta - \zeta^T\,d\tilde{\zeta})]\,,\\
u &= 2i\,e^{\frac{\tilde{K}+\hat{K}}{2}}\,Z^T\,(\overline{\mathcal{M}}\,d\zeta + d\tilde{\zeta})\,,\\
E &= i\,e^{\frac{\tilde{K}-\hat{K}}{2}}\,P\,N^{-1}\,(\overline{\mathcal{M}}\,d\zeta + d\tilde{\zeta})\,,\\
e &= P\,dZ\,,
\end{aligned} \tag{15}$$

where

$$e^{\tilde{K}} \;=\; \frac{1}{2\phi} = \frac{e^{2\varphi}}{2}, \;\;;\;\; e^{\hat{K}} = \frac{1}{2\bar{Z}NZ} = \frac{e^K}{2}\,, \tag{16}$$

and φ denotes the four dimensional dilaton. The matrices P and N are defined as follows:

$$P^{\underline{\sigma}}{}_0 \;=\; -e_\lambda{}^{\underline{\sigma}}Z^\lambda \;\;;\;\; P^{\underline{\sigma}}{}_\lambda = e_\lambda{}^{\underline{\sigma}}\;\;(\lambda = 1,\ldots,h_1)\,, \tag{17}$$

$$N_{\Lambda\Sigma} \;=\; \frac{1}{2}\mathrm{Re}(\frac{\partial^2\mathscr{F}}{\partial Z^\Lambda\partial Z^\Sigma})\,. \tag{18}$$

$e_\lambda{}^{\underline{\sigma}}$ being the vielbein of the special Kähler manifold embedded in the quaternionic manifold (the underlined indices being the rigid ones).

The metric on the quaternionic manifold reads [1]:

$$\begin{aligned}
ds^2 &= \bar{v}\,v + \bar{u}\,u + \bar{E}\,E + \bar{e}\,e =\\
&= K_{a\bar{b}}\,dz^a\,d\bar{z}^{\bar{b}} + \frac{1}{4\,\phi^2}\,(d\phi)^2 + \frac{1}{4\,\phi^2}\,(da + dV\times V)^2 - \frac{1}{2\,\phi}\,dV\,\mathscr{M}\,dV\,,
\end{aligned} \tag{19}$$

where the symplectic matrix $\mathscr{M}$ is defined as follows:

$$\mathscr{M} \;=\; \begin{pmatrix} 1\!\!1 & R\\ 0 & 1\!\!1\end{pmatrix}\begin{pmatrix} I & 0\\ 0 & I^{-1}\end{pmatrix}\begin{pmatrix} 1\!\!1 & 0\\ R & 1\!\!1\end{pmatrix} \;\;;\;\; R = \mathrm{Re}(\mathcal{M}),\, I = \mathrm{Im}(\mathcal{M})\,, \tag{20}$$

and V denotes the symplectic section: $V = \begin{pmatrix}\zeta^\Lambda\\ \tilde{\zeta}_\Lambda\end{pmatrix}$.

It is useful to rewrite the scalar potential in two equivalent ways:

$$\mathscr{V} \;=\; 4\,h_{uv}\,k^u_I\,k^v_J\,L^I\,\overline{L}^J + (U^{IJ} - 3\,L^I\overline{L}^J)\,k^u_I\,k^v_J\,\omega^x_u\,\omega^x_v\,, \tag{21}$$

$$\mathscr{V} \;=\; -\frac{1}{2}\,\mathrm{Im}\mathscr{N}^{-1IJ}\,k^u_I k^v_J\,\omega^x_u\,\omega^x_v + 4\,(h_{uv} - \omega^x_u\,\omega^x_v)\,k^u_I\,k^v_J\,L^I\,\overline{L}^J\,. \tag{22}$$

[2]Note that the Kähler potential K_{SK} and period matrix $\mathscr{N}_{SK}$, defined according to the conventions for special Kähler manifolds adopted for instance in [5], are related to the corresponding quantities in (14) in the following way: $e^{K_{SK}} = \frac{e^K}{4}$; $\mathscr{N}_{SK} = \frac{1}{4}\,\mathcal{M}$

In order to evaluate the expression on the right hand side of eq. (22) it is useful to compute the following quantity:

$$G_{IJ} = k_I^u\, k_J^v\, (h_{uv} - \omega_u^x\, \omega_v^x) = k_I^u\, k_J^v\, [\bar{v}\, v + \bar{u}\, u + \bar{E}\, E - (\bar{v}\, v + 4\, \bar{u}\, u)]_{uv}\,. \tag{23}$$

Using the following notation:

$$r_I = c_I + 2\, (e_I{}^\Lambda\, \tilde{\zeta}_\Lambda - e_{I\Lambda}\, \zeta^\Lambda)\ \ ;\ \ s_{I\Lambda} = e_{I\Lambda} + e_I{}^\Sigma\, \overline{\mathcal{M}}_{\Sigma\Lambda}\,, \tag{24}$$

we can express G_{IJ} as follows:

$$G_{IJ} = 2\, e^{\tilde{K}}\, \bar{s}_{I\Lambda}\, s_{J\Sigma}\, \left(\mathscr{U} - 3\, \mathcal{L}^\dagger\, \mathcal{L}\right)^{\Lambda\Sigma}\ \ ;\ \ \mathscr{U} = -\frac{I}{2} - \mathcal{L}^\dagger\, \mathcal{L}\ \ ;\ \ \mathcal{L} = e^{\frac{K}{2}}\, Z\,. \tag{25}$$

In deriving the above expression for G_{IJ} we made use of the following properties:

$$\begin{aligned} N^{-1} P^\dagger P N^{-1} &= e^K\, (-N^{-1} + \mathcal{L}^T\, \overline{\mathcal{L}})\,, \\ -\frac{I}{2} &= -N^{-1} + \mathcal{L}^T\, \overline{\mathcal{L}} + \mathcal{L}^\dagger\, \mathcal{L}\,. \end{aligned} \tag{26}$$

Now we can evaluate the two equivalent expressions for the scalar potential given in eqs. (21) and (22):

$$\begin{aligned} \mathscr{V} &= \overline{L}^I\, L^J\, \left[\frac{1}{\phi^2}\, (c_I + 2\, e_I \times V)\, (c_J + 2\, e_J \times V) - \frac{2}{\phi}\, e_I\, \mathscr{M}\, e_J\right] + \\ &\quad + \frac{1}{2\,\phi}\, (U - 3\, L^\dagger L)^{(IJ)}\, \left(\frac{1}{2\,\phi}\, r_I\, r_J + 8\, \bar{s}_{I\Lambda}\, s_{J\Sigma}\, \overline{\mathcal{L}}^\Lambda \mathcal{L}^\Sigma\right)\,, \end{aligned} \tag{27}$$

$$\begin{aligned} \mathscr{V} &= -\frac{1}{4\,\phi}\, \mathrm{Im}\mathscr{N}^{-1IJ}\, \left(\frac{1}{2\,\phi}\, r_I\, r_J + 8\, \bar{s}_{I\Lambda}\, s_{J\Sigma}\, \overline{\mathcal{L}}^\Lambda \mathcal{L}^\Sigma\right) + \\ &\quad + \frac{4}{\phi}\, \overline{L}^I\, L^J\, \bar{s}_{(I|\Lambda}\, s_{J)\Sigma}\, \left(\mathscr{U} - 3\, \mathcal{L}^\dagger\, \mathcal{L}\right)^{\Lambda\Sigma}\,, \end{aligned} \tag{28}$$

where we have introduced the following vectors: $e_I = \begin{pmatrix} e_I{}^\Lambda \\ e_{I\Lambda} \end{pmatrix}$. The first equation (27) is useful for those gaugings which involve just the graviphoton A_μ^0, e.g. Type IIA with NS flux or Type IIB on a half–flat "mirror" manifold. Indeed in these cases the term in the second line of (27) does not contribute for cubic special geometries in the vector multiplet sector since [5]:

$$(U - 3\, L^\dagger L)^{00} = 0\,. \tag{29}$$

Similarly the expression (28) is of particular use for those gaugings which involve only isometries $\Lambda = 0$, like for instance Type IIA on a half–flat manifold or Type IIB on the "mirror" manifold with NS flux since, for cubic special quaternionic geometries:

$$\left(\mathscr{U} - 3\, \mathcal{L}^\dagger\, \mathcal{L}\right)^{00} = 0\ \Rightarrow\ e^K = -\frac{1}{8}\, (I^{-1})^{00}. \tag{30}$$

In both cases the expressions for the potential reduce to those given in [16, 15].

3. The non–polynomial antisymmetric tensor Lagrangian

In this section we discuss the dualization of the special quaternionic σ–model when the $2\,h_1+3$ scalars, parametrizing the Heisenberg algebra are dualized into (2–form) antisymmetric tensors. Let us start with the Lagrangian for the Heisenberg scalars $(a, \zeta^\Lambda, \tilde{\zeta}_\Lambda)$ as given in [1]. In order to dualize the scalar a into the NS two form $B_{\mu\nu}$ we substitute da by the unconstrained 1–form η and add a further term:

$$\begin{aligned} L &= -\frac{1}{4\,\phi^2}\,(\eta + dV \times V) \wedge \star(\eta + dV \times V) + \frac{1}{2\,\phi}\,dV^A \wedge \star\,\mathscr{M}_{AB}\,dV^B + \\ & \quad H \wedge (\eta - da)\,. \end{aligned} \tag{31}$$

H being a 3–form Lagrange multiplier. Integrating out η we obtain the dual Lagrangian:

$$L = -\phi^2\,H \wedge \star H + \frac{1}{2\,\phi}\,dV^A \wedge \star\,\mathscr{M}_{AB}\,dV^B - dV \times dV \wedge B\,. \tag{32}$$

We see that upon dualization of the scalar a all the R–R scalars appear covered by derivatives, so that we may perform a further dualization. This is achieved by substituting in (32) dV^A by v^A, adding the Lagrange multiplier term $(v^A - dV^A) \wedge G_A$, G_A being a set of 3–forms, and integrating out v^A. We find:

$$\frac{1}{\phi}\,\mathscr{M}_{AB} \star v^B + f_{AB}{}^0\,v^B \wedge B_0 + G_A = 0\,. \tag{33}$$

Denoting by $G^\mu_A = \epsilon^{\mu\nu\rho\sigma}\,G_{\nu\rho\sigma\,A}$, the above equation in tensor components reads:

$$G_{\mu A} = K_{A\mu,\,B\nu}\,v^{\nu B}\,, \tag{34}$$

$$K_{\mu A,\,\nu B} = \Delta_{AB}\,g_{\mu\nu} - f_{AB}{}^0\,\epsilon_{\mu\nu\rho\sigma}\,B_0^{\rho\sigma}\,, \tag{35}$$

$$\Delta_{AB} = \frac{1}{\phi}\,\mathscr{M}_{AB}\,, \tag{36}$$

where $B_{\mu\nu\,0} = B_{\mu\nu}$. Upon implementation of eq. (33) the dual Lagrangian can be computed to be:

$$\begin{aligned} \mathscr{L}_D &= -\phi^2\,H \wedge \star H + \frac{1}{2}\,G_{\mu A}\,\tilde{K}^{\mu A,\,\nu B}\,G_{\nu B}\,, \\ K_{\mu A,\,\nu B}\,\tilde{K}^{B\nu,\,C\rho} &= \delta^C_A\,\delta^\rho_\mu\,. \end{aligned} \tag{37}$$

This Lagrangian has the form of the non–polynomial model discussed in [6]. Using the above notations eq. (34) can be inverted to give:

$$v^{\mu A} = \tilde{K}^{A\mu,\,B\nu}\,G_{\nu B}\,. \tag{38}$$

We may now derive the equations of motion and the invariance of the Lagrangian (37). In order to compute the variation of the Lagrangian corresponding to an arbitrary variation

of the tensor fields B, B_A, it is convenient to express the various terms in the variation in terms of the composite fields v^A using equation (34). One then obtains:

$$\delta\mathscr{L}_D = \mathscr{F}^0 \wedge \delta B_0 + \mathscr{F}^A \wedge \delta B_A\,, \tag{39}$$

$$\mathscr{F}^0 = dv^0 + \frac{1}{2} f_{AB}{}^0\, v^A \wedge v^B = dv^0 - \mathbb{C}_{AB}\, v^A \wedge v^B\ ;\quad \mathscr{F}^A = dv^A\,. \tag{40}$$

The equations of motion therefore read:

$$\mathscr{F}^0 = \mathscr{F}^A = 0\,. \tag{41}$$

Furthermore, using eq. (39), one can check that the Lagrangian (37) is invariant, up to total derivatives, under the following tensor–gauge transformation:

$$\delta B_0 = D\xi_0 = d\xi_0\ ;\quad \delta B_A = D\xi_a = d\xi_A + f_{AB}{}^0\, v^B\, \xi_0 = d\xi_A - 2\,\mathbb{C}_{AB}\; v^B\, \xi_0\,. \tag{42}$$

Following [6], the dual Lagrangian (37) can be written in a polynomial form by adopting a first order formalism in which the fields v^A are treated as independent of B_A. In this formulation the Lagrangian can be written in the following form:

$$\mathscr{L}_D^{(1)} = -\phi^2\, H \wedge \star H + \frac{1}{2}\Delta_{AB}\, v^A \wedge \star v^B + B_0 \wedge \mathscr{F}^0 + B_A \wedge \mathscr{F}^A\,. \tag{43}$$

Indeed the equation of motion for v^A yields the relation (34) which, if substituted in $\mathscr{L}_D^{(1)}$, reproduces the second order Lagrangian $\mathscr{L}_D$. As already noted in [6], the tensor–gauge invariance of the first order Lagrangian is much simpler since the corresponding variations are now written in the following form:

$$\delta B_0 = d\xi_0\ ;\ \delta B_A = D\xi_A\ ;\quad \delta v^A = 0\,, \tag{44}$$

and such gauge transformations are of course abelian.

The $N = 2$ supersymmetric completion of the non–polynomial Lagrangian (37) is not straightforward and should involve the self–coupling of the h_1 double–tensor multiplet with a triple–tensor multiplet (the latter originating from the universal hypermultiplet [24]).

Gauging of the model in the presence of RR fluxes.

We can now try to couple the model to $h_2 + 1$ vector fields through electric and magnetic charges $e_I{}^m = \{e_I{}^0, e_I{}^A\}$, $m^{Im} = \{m^{I0}, m^{IA}\}$. Introducing only R–R charges $e_I{}^0$, m^{I0} poses no problem and the gauged Lagrangian has the form:

$$\begin{aligned}\mathscr{L}_g^{(1)} = {}& -\phi^2\, H \wedge \star H + \frac{1}{2}\Delta_{AB}\, v^A \wedge \star v^B + B_0 \wedge \mathscr{F}^0 + B_A \wedge \mathscr{F}^A - \\ & -\frac{1}{2}\,\mathrm{Im}(\mathscr{N})_{IJ}\,(dA^I + m^{I0}\, B_0) \wedge \star(dA^I + m^{I0}\, B_0) + \\ & +\mathrm{Re}(\mathscr{N})_{IJ}\,(dA^I + m^{I0}\, B_0) \wedge (dA^I + m^{I0}\, B_0) - e_I{}^0\, B_0 \wedge (dA^I + \frac{1}{2} m^{I0}\, B_0) \end{aligned} \tag{45}$$

(46)

If one prefers to work in the second order formalism, it suffices to use for v^A the expression derived from eq. (34), all the terms not containing v in (45) being left unchanged. The Lagrangian (45) is clearly invariant under the following tensor–gauge transformation:

$$\delta B_0 \;=\; d\xi_0 \;\;;\;\; \delta B_A = D\xi_A \;\;;\;\; \delta A^I = -m^{I0}\, d\xi_0 \;\;;\;\; \delta v^A = 0\,, \tag{47}$$

and vector–gauge transformations:

$$\delta B_0 \;=\; \delta B_A = \delta v^A = 0 \;\;;\;\; \delta A^I = d\lambda^I\,. \tag{48}$$

The introduction of the remaining charges is more problematic and is left to future investigations.

4. Concluding remarks

In this paper we have investigated different aspects of Calabi–Yau compactifications to four dimensions in the presence of fluxes and their interpretation in terms of $N = 2$ massive supergravities. When the quaternionic manifold is of special type, as it occurs in $N = 2$ theories encompassing these compactifications, the scalar potential in the presence of cubic special geometries has a dual no–scale structure for certain choices of fluxes. These choices allow to reproduce on general grounds some computations of the scalar potential coming from half–flat manifolds or theories with NS flux [15, 16, 17].

In an attempt to introduce further fluxes, corresponding to magnetic charges, it is natural to consider a dual form of the special quaternionic geometry in which the Heisenberg scalars are replaced by antisymmetric tensors. This results in a non–polynomial tensor theory à la Freedman and Townsend [6] which has a manifest symplectic invariance under the tensor–gauge transformation given in (44) and (42). We note that the non–polynomial antisymmetric tensor theory only contains as scalars the dilaton φ in addition to NS Calabi–Yau h_1 complex deformations encoded in the special geometry of the original special quaternionic manifold. When this system is coupled to vector multiplets, a manifest $\mathrm{Sp}(2\,h_1+2) \times \mathrm{Sp}(2\,h_2+2)$ of the mirror special geometries is exhibited. The possibility of adding electric and magnetic fluxes in this framework will be discussed elsewhere.

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